ILLINOIS LICENSURE TESTING SYSTEM

FIELD 208: MATHEMATICS

TEST FRAMEWORK

January 2017

Effective beginning May 14, 2018

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Mathematical Practices, Technology, and Disciplinary Literacy
Number Systems and Operations
Functions and Algebra
Differential and Integral Calculus
Measurement and Geometry
Statistics, Probability, and Discrete Mathematics

SUBAREA I—MATHEMATICAL PRACTICES, TECHNOLOGY, AND DISCIPLINARY LITERACY

0001 Understand mathematical practices.

For example:

- Apply a variety of strategies (e.g., drawing diagrams, working backward, identifying mathematical structures, recognizing patterns).
- Generalize the processes used to solve problems and extend them to other problem situations.
- Select appropriate models to analyze and understand situations to improve decision making.
- Justify applications of concepts, procedures, and theorems in a given situation.
- Develop conjectures and evaluate their validity, and critique the reasoning of others.
- Demonstrate knowledge of how to communicate mathematical knowledge using precise mathematical vocabulary, visual representations, and symbols.
- Identify and apply connections between mathematical domains (e.g., the distance formula in coordinate geometry and the Pythagorean Theorem, geometric interpretation of the integral, applications of matrices to geometric figures).
- Demonstrate knowledge of mathematical connections to other disciplines (e.g., rate of change as applied to business, economics, physics, chemistry, and biology; geometry as applied to carpentry and construction).

0002 Understand how to select, integrate, and use appropriate technologies in the mathematics classroom.

For example**:**

- Demonstrate understanding of the capabilities and benefits of current technologies in the content area of mathematics.
- Select appropriate technology (e.g., spreadsheet, graphing calculator, manipulatives) to analyze concepts and solve problems throughout the mathematics curriculum.
- Demonstrate understanding of how to integrate appropriate technology in mathematics instruction and assessment.
- Develop and apply algorithms to analyze concepts and solve problems using technology.
- Use appropriate technology to individualize and differentiate mathematics instruction and assessment.

0003 Understand the process of reading, and apply knowledge of strategies for promoting students' reading development in the mathematics classroom.

For example:

- Demonstrate knowledge of the reading process (e.g., the construction of meaning through interactions between a reader's prior knowledge, information in the text, and the purpose of the reading situation), and apply knowledge of strategies for integrating language arts into mathematics instruction to support students' reading and concept development (e.g., providing purposeful opportunities for students to read, write about, and discuss content in order to improve their understanding).
- Apply knowledge of strategies that foster reading in the mathematics classroom (e.g., incorporating literature for adolescents in the curriculum; encouraging students' independent reading, research, and inquiry related to mathematics).
- Demonstrate knowledge of vocabulary's role in supporting students' reading comprehension and concept development, and apply strategies for promoting students' mathematics vocabulary development (e.g., recognizing structural and/or meaning-based relationships between words, using context clues, distinguishing denotative and connotative meanings of words, interpreting idioms and figurative language, consulting specialized reference materials).

- Apply knowledge of strategies for preparing students to read text effectively and for teaching and modeling the use of comprehension strategies before, during, and after reading, including strategies that promote close reading (e.g., breaking down complex sentences, monitoring for comprehension to correct confusions and misunderstandings that arise during reading).
- Apply knowledge of strategies for developing students' ability to comprehend and critically analyze discipline-specific texts, including recognizing organizational patterns unique to informational texts; using graphic organizers to analyze and recall information from texts; analyzing and summarizing an author's argument, claims, evidence, and point of view; and synthesizing multiple sources of information presented in different media or formats.
- Apply knowledge of strategies for evaluating, selecting, modifying, and designing reading materials appropriate to academic tasks and students' reading abilities (e.g., analyzing instructional materials in terms of readability, content, length, format, illustrations).
- Apply knowledge of strategies for providing continuous monitoring of students' reading progress through observations, work samples, and various informal assessments and for differentiating mathematics instruction to address students' assessed reading needs.

SUBAREA II—NUMBER SYSTEMS AND OPERATIONS

0004 Understand the real number system and its operations.

For example:

- Demonstrate knowledge of the structure and properties of the real number system.
- Solve problems involving prime and composite numbers, least common multiples, greatest common factors, and modular arithmetic.
- Solve problems involving integers, fractions, decimals, percents, powers, and rational exponents.
- Apply ratios and proportional reasoning to represent and solve problems.
- Apply knowledge of dimensional analysis to model and solve problems using appropriate units.
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

0005 Understand the properties of complex numbers and linear algebra.

For example:

- Perform arithmetic operations with complex numbers and their conjugates.
- Represent complex numbers and their operations in rectangular and polar forms on the complex plane.
- Apply knowledge of the fundamental theorem of algebra to solve quadratic equations with real coefficients that have complex solutions.
- Apply basic properties of vectors and vector arithmetic to model and solve problems.
- Apply basic properties of matrices and matrix arithmetic to model and solve problems (e.g., solve a system of two linear equations).
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

SUBAREA III—FUNCTIONS AND ALGEBRA

0006 Understand algebraic techniques.

For example:

- Translate between word situations and algebraic sentences.
- Identify algebraic expressions or equations that represent mathematical situations (e.g., symbolically, numerically, graphically).
- Identify, complete, and extend patterns, sequences, and series and analyze their properties and algebraic representations.
- Apply properties of real numbers in algebraic contexts to manipulate and simplify algebraic expressions and solve equations.
- Justify algebraic techniques using properties of the real number system.
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and application.

0007 Understand functions and the properties of linear relations and functions.

For example:

- Apply function notation and analyze the properties of relations and functions, including the domain and range of linear, polynomial, absolute value, radical, rational, exponential, logarithmic, and trigonometric functions.
- Build new functions from existing functions (e.g., transformations, inverses, compositions).
- Represent linear relations and functions in symbolic, numeric, graphic, and verbal forms.
- Recognize and apply slopes and intercepts to construct, analyze, interpret, and graph linear equations and inequalities.
- Apply linear relations, functions, and systems to model and solve problems.
- Represent and solve systems of linear equations and inequalities both graphically and algebraically (e.g., compare and contrast different solution methods).
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

0008 Understand quadratic functions and conic sections.

For example:

- Represent quadratic functions and relations in symbolic, numeric, graphic, and verbal forms.
- Recognize and apply properties of symmetry, roots, intercepts, and vertices to construct, analyze, and interpret quadratic relations and their graphs.
- Solve quadratic equations and inequalities and systems of equations and inequalities both graphically and algebraically.
- Apply quadratic relations, functions, and systems to model and find appropriate solution(s) to problems.
- Recognize and apply the properties of hyperbolas, parabolas, circles, and ellipses to model and solve problems.
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

0009 Understand polynomial, absolute value, radical, and rational functions and inequalities.

For example:

- Represent polynomial, absolute value, radical, and rational functions and relations in symbolic, numeric, graphic, and verbal forms.
- Recognize and apply symmetry, roots, intercepts, critical points, asymptotes, and vertices to construct, analyze, and interpret polynomial, absolute value, radical, and rational functions and inequalities and their graphs.
- Recognize and apply the properties of polynomial, absolute value, radical, and rational functions and equations to solve problems.
- Solve polynomial, absolute value, radical, and rational equations and inequalities and systems of equations and inequalities graphically, algebraically, and numerically.
- Apply polynomial, absolute value, radical, and rational relations, functions, and systems to model and solve problems.
- Recognize and apply the algebraic properties of polynomial and rational functions (e.g., factoring, partial fractions).
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

0010 Understand exponential and logarithmic functions.

For example:

- Recognize the relationship between exponential and logarithmic functions.
- Represent exponential and logarithmic functions in symbolic, numeric, graphic, and verbal forms.
- Recognize and apply properties of symmetry, roots, intercepts, critical points, and asymptotes to construct, analyze, and interpret exponential and logarithmic functions and inequalities and their graphs.
- Apply the equations of exponential and logarithmic functions and systems to make predictions and model and solve problems (e.g., compound interest, exponential growth and decay, logistics).
- Solve exponential and logarithmic equations and inequalities and systems of equations and inequalities both graphically and algebraically.
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

0011 Understand trigonometric functions.

For example:

- Apply trigonometric functions to solve problems involving the unit circle.
- Analyze relationships between trigonometric functions and their graphs.
- Use trigonometric functions to model periodic phenomena.
- Manipulate trigonometric expressions and solve equations.
- Identify or verify trigonometric identities using algebraic techniques.
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

SUBAREA IV—DIFFERENTIAL AND INTEGRAL CALCULUS

0012 Understand principles of differential calculus.

For example:

- Apply the concept of a limit to analyze properties of functions (e.g., continuity, asymptotes) and series.
- Demonstrate conceptual knowledge of the derivative and its role in the historical development of calculus.
- Find the derivative of a function and interpret its meaning.
- Apply differentiation to analyze the graph of a function (e.g., maxima, concavity).
- Apply principles of differential calculus to model and solve problems (e.g., rates of change, optimization, analyzing functions, related rates).
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

0013 Understand principles of integral calculus.

For example:

- Demonstrate conceptual knowledge of the integral.
- Calculate the integrals of functions.
- Apply the fundamental theorem of calculus to model and solve problems.
- Apply principles of integral calculus to solve problems (e.g., finding areas and volumes, describing the motion of an object).
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

SUBAREA V—MEASUREMENT AND GEOMETRY

0014 Understand principles and applications of measurement.

For example:

- Convert measurements within various systems of measurement (e.g., English, metric).
- Solve problems involving time, temperature, degree and radian measures of angles, weight, and mass.
- Use nonlinear measuring scales (e.g., Richter, decibel, pH) to solve practical problems.
- Explain how changing one measure of a multidimensional object may affect other measures.
- Apply measurement formulas (e.g., length, area, volume) to regular and irregular shapes, regions, and solids.
- Solve problems involving indirect measurement (e.g., trigonometric ratios, proportional reasoning).
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

0015 Understand Euclidean geometry.

For example:

- Demonstrate understanding of points, lines, angles, planes, and space and their geometric applications.
- Apply definitions, axioms, principles, and theorems of Euclidean geometry to develop or analyze different types of proofs (e.g., direct, indirect, flow, paragraph).
- Solve problems in Euclidean geometry (e.g., justify geometric constructions).
- Apply properties of circles and other two- and three-dimensional figures to solve mathematical and real-world problems.
- Apply concepts of similarity and congruence to analyze the properties and compare the measures (e.g., perimeter, area, volume) of two- and three-dimensional figures.
- Translate between two- and three-dimensional representations of geometric figures (e.g., cross sections, nets, projections, perspective drawings).
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

0016 Understand coordinate and transformational geometry.

For example:

- Use coordinate geometry methods (e.g., distance formula, slope) to prove theorems and analyze geometric figures.
- Use the slope criteria for parallel and perpendicular lines to find the equations of lines and solve geometric problems.
- Demonstrate knowledge of congruence, similarity (including scale and size changes), and symmetry from the perspective of any geometric transformation.
- Analyze transformations (i.e., reflections, rotations, translations, dilations) in the coordinate plane.
- Use two- and three-dimensional coordinate systems to represent and analyze geometric figures.
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

SUBAREA VI—STATISTICS, PROBABILITY, AND DISCRETE MATHEMATICS

0017 Understand the process of collecting, organizing, and representing data.

For example:

- Demonstrate knowledge of appropriate sampling techniques that collect representative data and avoid bias.
- Calculate and interpret measures of central tendency (e.g., mean, median, mode) and variation (e.g., range, standard deviation) to characterize a given set of data.
- Select and justify an appropriate measure of central tendency from given data.
- Select appropriate representations and interpretations of data using a variety of graphs (e.g., bar graphs, scatterplots, box plots, stem-and-leaf diagrams, histograms).
- Estimate population percentages in normal distributions, given the mean and standard deviation.
- Analyze data presented in a two-way frequency table or scatterplot (e.g., draw conclusions, interpret a correlation coefficient, identify a line of best fit).
- Evaluate inferences and claims based on sample data.
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.

0018 Understand probability and discrete mathematics.

For example:

- Demonstrate knowledge of basic set theory (e.g., unions, intersections) and logic (e.g., if/then, if and only if).
- Apply various counting strategies (e.g., combinations, permutations) to represent and solve problems, including those involving probability.
- Find probabilities using a variety of probability techniques (e.g., addition and multiplication rules, determining sample spaces) for different kinds of events (e.g., independent, conditional).
- Apply knowledge of expected value to analyze problem situations (e.g., fairness of games, lotteries) and determine the probability of events.
- Use a variety of graphic representations to represent situations and determine probabilities (e.g., Venn diagrams, trees).
- Select or analyze simulations that model simple theoretical and experimental probabilities.
- Demonstrate understanding of instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and enable students to develop understanding of mathematical practices, concepts, skills, and applications.